

**Note****COMPUTER-DETERMINED KINETIC PARAMETERS FROM TG CURVES. PART XIV**

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The present authors recently presented computer programs which could be used to analyze isothermal TG (ITG) data. In this regard, computer procedures were devised for the estimation of: activation energy ( $E$ ) [1], and mechanism and rate constant [2–5]. None of the preceding ITG procedures allowed for the determination of both  $E$  and mechanism concurrently. In this paper, a computer procedure (CP) will be presented which will allow both  $E$  and mechanism to be estimated simultaneously. This CP was tested utilizing theoretical ITG data generated by means of a computer.

## THEORY

For an isothermal unimolecular solid-state decomposition, we may write

$$g(\alpha) = kt \quad (1)$$

where  $t$  = reaction time,  $g(\alpha) = \int_0^\alpha d\alpha/f(\alpha)$ ,  $\alpha$  = fractional conversion, and  $f(\alpha)$  = some function of  $\alpha$  which is theoretically possible. From eqn. (1), the following can be readily obtained

$$\ln[g(\alpha)/t] = -E/RT + \ln(Z) \quad (2)$$

where  $Z$  = pre-exponential factor, and  $T$  = temperature (K).

In order to employ eqn. (2) for the purposes previously mentioned, several ITG runs are carried out at various temperatures. Then for various time periods ( $t$ ), various values of  $\alpha$  are obtained for each temperature. In this paper, ten possible theoretical mechanisms will be tested by means of a computer. Equation (2) was employed, and for each mechanism, a least-squares procedure carried out for each particular value of  $t$  to afford corresponding values of slope ( $E/R$ ) and intercept ( $\ln(Z)$ ). Then for all the  $t$ -values utilized, averages of  $E/R$  and  $\ln(Z)$  are estimated for each mechanism. Further, the mean deviation (DIFF) for  $\ln(Z)$  is obtained for each mechanism. The most probable mechanism (and  $E$ -value) is considered to be the one whose DIFF possesses the lowest value.

## RESULTS AND DISCUSSION

The following ten theoretical mechanisms were examined to ascertain which one of them best conformed to the isothermal TG data: A4, A3, and A2 (random nucleation, Avrami–Erofeev equations); R2 and R3 (phase boundary reaction, cylindrical and spherical symmetry, respectively); F1 (random nucleation, one nucleus per particle); D1, D2, D3, and D4 (corresponding to 1-dimensional diffusion; 2-dimensional diffusion, cylindrical symmetry; 3-dimensional diffusion, Jander spherical symmetry; and 3-dimensional diffusion, Ginstling–Brounshtein spherical symmetry).

Theoretical computer values of  $\alpha$  were generated for various  $t$ - and  $T$ -values for the mechanisms: A2, F1, R2 and D3. Thus, for example, for the R2-mechanism, values were obtained from the expression;  $\alpha = 1 - B^2$  where  $B = 1 - 0.5Zt \exp(-E/RT)$  (0.5 is an integration factor). For the four preceding mechanisms, the following values of  $Z$  and  $E$  (kcal mol<sup>-1</sup>) were employed, respectively:  $3 \times 10^{10}$ , 24.9;  $1 \times 10^{13}$ , 30.0;  $1 \times 10^{13}$ , 30.0;  $1 \times 10^{14}$ , 35.0.

The seven specific  $t$ -values of 20, 30, 40, 50, 60, 70, and 80 were employed for all the mechanisms tested. In the Appendix a computer printout of the various ITG data employed is shown along with results of their computer analysis. In each of lines 525, 540, 550, and 560 are listed the five  $T$ -values (K) used in testing for the four mechanisms. In each of lines 535, 545, 555, and 565 are listed corresponding values of  $\alpha$ . Thus, for example, in line 535, each of the first five  $\alpha$  values correspond to each of the  $T$ -values listed in line 525 for the particular value of  $t = 20$ . Then, the next group of five  $\alpha$  values also corresponds to each of the  $T$ -values in line 525, but for the particular value of  $t = 30$ , etc., up to  $t = 80$ .

From the Appendix, it can be observed that there is good agreement between calculated and expected results, i.e., probable mechanism and  $E$ , when values of  $\alpha$  possess four significant figures (SF). This agreement was still good when three SF were used for  $\alpha$ . However, when only two SF were employed for  $\alpha$ , there was little agreement; thus, the probable mechanism and its corresponding value of  $E$  (kcal mol<sup>-1</sup>) are given in the following for the mechanisms A2, F1, R2, D3, respectively: A4, 12.5; A2, 14.9; R2, 30.0; A4, 5.6. From the preceding, it can be perceived that there were changes in mechanism (and  $E$ ) for all but the R2-mechanism. Thus, in utilizing the computer procedure described above, it is necessary that  $\alpha$ -values possess an accuracy of 3 or more SF to obtain reliable results.

## APPENDIX

A computer printout of various theoretical ITG data (lines 525-565, see the text for explanation) and the results of their computer analysis for the most probable mechanisms and corresponding  $E$ -values.

525 DATA 440,445,450,455,460: REM TEMPS. USED (K) FOR A2-MECH. AND DATA IN #525

535 DATA .01632,.03073,.05668,.1020,.1779,.03635,.06781,.1230,.2150,.3564,.06370,.1174,.2082,.3498,.5431,.09774,.1772,.3056,.4896,.7059,.1377,.2449,.4085,.6204,.8284,.1826,.3177,.5107,.7324,.9092,.2315,.3931,.6069,.8213,.9564

| MECHNSM.                | DIFF.    | E (K/M) |
|-------------------------|----------|---------|
| A4/(-LN(1-A))^(1/4)     | .1495    | 12.4    |
| A3/(-LN(1-A))^(1/3)     | .12215   | 16.6    |
| A2/(-LN(1-A))^(1/2)     | 1.06E-03 | 24.9    |
| R2/1-(1-A)^(1/2)        | 4.15988  | 43      |
| R3/1-(1-A)^(1/3)        | 2.92352  | 45.3    |
| F1/-LN(1-A)             | .40485   | 49.8    |
| D1/A^2                  | 12.08173 | 78.6    |
| D2/A+(1-A)LN(1-A)       | 8.94984  | 84.4    |
| D4/1-(2A/3)-(1-A)^(2/3) | 7.43564  | 86.8    |
| D3/(1-(1-A)^(1/3))^2    | 4.26744  | 91.7    |

PROB. MECHNSM.: A2/(-LN(1-A))^(1/2) WITH DIFF.= 1.056E-03 & E= 24.9 KCAL/MOL

540 DATA 430,435,440,445,450: REM TEMPS. USED (K) FOR F1-MECH. AND DATA IN #545

545 DATA .1066,.1553,.2215,.3080,.4154,.1556,.2237,.3131,.4244,.553,.2019,.2865,.394,.5212,.6582,.2456,.3443,.4653,.6017,.7387,.287,.3973,.5282,.6687,.8002,.3261,.4461,.5838,.7244,.8472,.363,.4909,.6327,.7707,.8832

| MECHNSM.                | DIFF.    | E (K/M) |
|-------------------------|----------|---------|
| A4/(-LN(1-A))^(1/4)     | .22383   | 7.5     |
| A3/(-LN(1-A))^(1/3)     | .24343   | 10      |
| A2/(-LN(1-A))^(1/2)     | .20637   | 15      |
| R2/1-(1-A)^(1/2)        | 2.14328  | 25      |
| R3/1-(1-A)^(1/3)        | 1.46174  | 26.7    |
| F1/-LN(1-A)             | 1.65E-03 | 30      |
| D1/A^2                  | 5.77323  | 42.9    |
| D2/A+(1-A)LN(1-A)       | 4.29803  | 47.7    |
| D4/1-(2A/3)-(1-A)^(2/3) | 3.57993  | 49.7    |
| D3/(1-(1-A)^(1/3))^2    | 2.10612  | 53.6    |

PROB. MECHNSM.: F1/-LN(1-A) WITH DIFF.= 1.648E-03 & E= 30 KCAL/MOL

550 DATA 425,430,435,440,445: REM TEMPS. USED (K) FOR R2-MECH. AND DATA IN #555

555 DATA .0732,.1096,.1617,.2347,.3343,.1088,.162,.2372,.3404,.4761,.1436  
 .2128,.3091,.4381,.6009,.1778,.262,.3775,.5281,.7087,.2113,.3096,.44  
 23,.6102,.7996,.244,.3557,.5035,.6844,.8735,.2761,.4001,.5612,.7508,.  
 9305

| MECHNSM.                | DIFF.    | E (K/M) |
|-------------------------|----------|---------|
| A4/(-LN(1-A))^(1/4)     | .4581    | 9       |
| A3/(-LN(1-A))^(1/3)     | .71193   | 11.9    |
| A2/(-LN(1-A))^(1/2)     | 1.28469  | 17.8    |
| R2/1-(1-A)^(1/2)        | 2.24E-03 | 30      |
| R3/1-(1-A)^(1/3)        | .97828   | 31.7    |
| F1/-LN(1-A)             | 2.98112  | 35.1    |
| D1/A^2                  | 3.42727  | 51.9    |
| D2/A+(1-A)LN(1-A)       | 1.27576  | 56.9    |
| D4/1-(2A/3)-(1-A)^(2/3) | .24055   | 58.9    |
| D3/(1-(1-A)^(1/3))^2    | 1.93794  | 63      |

PROB. MECHNSM.: R2/1-(1-A)^(1/2) WITH DIFF.= 2.245E-03 & E= 30 KCAL/MOL

560 DATA 455,460,465,470,475: REM TEMPS. USED (K) FOR D3-MECH. AND DATA IN #565

565 DATA .3711,.4421,.5203,.604,.6905,.4393,.519,.6045,.6929,.78,.4929,.  
 5781,.6674,.7567,.8404,.5371,.6259,.7169,.8049,.8835,.5749,.666,.7572  
 .8426,.9151,.6077,.7002,.7907,.8727,.9385,.6367,.7298,.819,.897,.956  
 1

| MECHNSM.                | DIFF.    | E (K/M) |
|-------------------------|----------|---------|
| A4/(-LN(1-A))^(1/4)     | .06045   | 5.6     |
| A3/(-LN(1-A))^(1/3)     | .16462   | 7.5     |
| A2/(-LN(1-A))^(1/2)     | .43694   | 11.2    |
| R2/1-(1-A)^(1/2)        | .83375   | 15.5    |
| R3/1-(1-A)^(1/3)        | .21249   | 17.5    |
| F1/-LN(1-A)             | 1.26885  | 22.1    |
| D1/A^2                  | 2.98315  | 21.7    |
| D2/A+(1-A)LN(1-A)       | 2.06962  | 27.1    |
| D4/1-(2A/3)-(1-A)^(2/3) | 1.42718  | 29.7    |
| D3/(1-(1-A)^(1/3))^2    | 3.95E-03 | 35      |

PROB. MECHNSM.: D3/(1-(1-A)^(1/3))^2 WITH DIFF.= 3.955E-03 & E= 35 KCAL/MOL

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